

## Math 1151

### Spring 2003 Final Exam Problems

This exam contains 12 multiple-choice questions, worth 8 points each, and 8 written problems, worth various amounts of points, for a total of 200 points.

[1] Find the exact value of the expression

$$\sin(77^\circ) \cos(47^\circ) - \cos(77^\circ) \sin(47^\circ)$$

- a)  $\frac{1}{2}$
- b)  $-\frac{1}{2}$
- c) 0
- d)  $\frac{\sqrt{3}}{2}$
- e)  $-\frac{\sqrt{3}}{2}$

[2] Find the exact value of the expression

$$\sin\left(\cos^{-1}\left(\frac{5}{13}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$$

- a)  $\frac{63}{65}$
- b)  $\frac{20}{65}$
- c) 0
- d)  $\frac{33}{65}$
- e)  $\frac{77}{65}$

[3] The polar form of  $z = (1 - i)^3$  is:

- a)  $\sqrt{8}\left(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4}\right)$
- b)  $2\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$
- c)  $2\sqrt{2}\left(\cos \frac{5\pi}{4} - i \sin \frac{5\pi}{4}\right)$
- d)  $\sqrt{8}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$
- e) None of the above

[4] What is the correct value of the sum

$$\sum_{k=1}^{12} \left( \frac{3}{2}k^3 - 1 \right) ?$$

- a) 9824
- b) 1342
- c) 9114
- d) 9672
- e) None of the above

[5] What is the sum equal to ?

$$\sum_{k=1}^{11} 2(-3)^{k-1}$$

- a) None of the following
- b)  $2 \cdot \frac{1-3^{11}}{1-(-3)}$
- c)  $2 \cdot \frac{1-(-3)^{11}}{1-(-3)}$
- d)  $2 \cdot \frac{1-3^{11}}{1-3}$
- e)  $-2 \cdot \frac{1-3^{11}}{1-3}$

[6] What is the equation of an ellipse with foci at  $(-4, 2)$  and  $(-4, 8)$  and vertex at  $(-4, 10)$  ?  
(It may help to draw a picture!)

- a)  $\frac{(x-4)^2}{16} + \frac{(y-5)^2}{25} = 1$
- b)  $\frac{(x+4)^2}{16} + \frac{(y-5)^2}{25} = 1$
- c)  $\frac{(x-4)^2}{25} + \frac{(y-5)^2}{16} = 1$
- d)  $\frac{(x+4)^2}{25} + \frac{(y-5)^2}{16} = 1$
- e)  $\frac{(x-4)^2}{25} - \frac{(y-5)^2}{16} = 1$

[7] What is the area of a triangle which has 2 adjacent sides equal to 5 and 4 respectively, and the angle formed by these sides is  $30^\circ$  ?

- a) 5
- b) 10
- c)  $10\sqrt{3}$
- d)  $5\sqrt{3}$
- e) None of the above

[8] Determine which of the four sequences below are arithmetic.

- I) 4, 12, 36, 108, 972, ...
- II) 2, 4, 6, 10, 12, ...
- III) 2, -3, -8, -13, -18, ...
- IV) 2, -1, -4, -7, -10, ...

- a) II, III and IV only
- b) III and IV only
- c) II and III only
- d) I only
- e) None of them

[9] What is the sum equal to ?

$$\sum_{k=1}^{\infty} 2 \left( -\frac{1}{3} \right)^{k-1}$$

- a) 1.5
- b) Does not exist
- c) -0.6667
- d) -1
- e) 3

[10] Consider the equation  $x^2 = 4y$ . In polar coordinates, this equation simplifies to

a)  $r \sin^2 \theta = 4 \cos \theta$

b)  $4 \cos^2 \theta = r \sin \theta$

c)  $4 \sin^2 \theta = r \cos \theta$

d)  $r \cos^2 \theta = 4 \sin \theta$

e) None of the above

[11] Find a third-degree polynomial function with real coefficients and with zeros 1 and  $3 + i$ .

a)  $x^3 - 7x^2 + 4x - 10$

b)  $x^3 - 5x^2 + 4x + 10$

c)  $x^3 + 7x^2 + 4x - 10$

d)  $x^3 + 7x^2 + 16x + 10$

e)  $x^3 - 7x^2 + 16x - 10$

[12] What is the period of the function

$$f(x) = -\frac{1}{2} \sin(4x + 3\pi) \quad ?$$

a) 4

b)  $\pi$

c)  $\frac{\pi}{2}$

d)  $2\pi$

e)  $\frac{3}{4}\pi$

- [13] Solve the following trigonometric equation for  $0 \leq \theta < 2\pi$ :

$$1 + \sqrt{3} \cos \theta + \cos(2\theta) = 0$$

[ 14 pts.]

- [14] Find the coordinates of the center and vertices of the conic section and draw them together with the curve. Your graph need not be perfect, but you should use the best techniques shown in class to make it as accurate as possible.

$$-4x^2 + y^2 - 16x - 2y - 19 = 0$$

[ 14 pts.]

- [15] Given the polynomial  $f(x) = x^4 - 8x^3 + 16x^2 + 8x - 17$ .

- a) (6 points) Write  $f(x)$  in factored form.  
b) (8 points) Find all of the complex zeros.

[ 14 pts.]

- [16] Graph the following system of inequalities.

$$\begin{cases} x & \geq 0 \\ y & \geq 0 \\ x^2 + y^2 & \leq 4 \\ (x-1)^2 + y^2 & \leq 4 \end{cases}$$

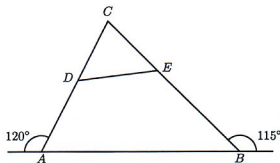
[ 12 pts.]

- [17] Prove the trigonometric identity:

$$1 - 8 \sin^2(\theta) \cos^2(\theta) = \cos(4\theta)$$

[ 12 pts.]

[18] In the triangle below in addition to the angles mentioned in the picture we know that  $AB = 3$ ,  $CD = \frac{1}{4}$ ,  $CE = \frac{1}{2}$ . Find the length  $AD + DE + EB$ . (The picture might not be perfectly according with the data. Use the given data in the problem.)



[14 pts.]

[19] Given the complex number  $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot i$ .

- (4 points) Find the polar form of  $z$
- (4 points) Find  $z^3$
- (6 points) Find all complex 3 roots of  $z$ .

Give all the answers in polar form.

[14 pts.]

[20] If  $\sin \alpha = \frac{5}{6}$  and  $\cos \beta = \frac{2}{7}$ , where  $\frac{\pi}{2} \leq \alpha \leq \pi$  and  $-\frac{\pi}{2} \leq \beta \leq 0$ , find the exact value of  $\cos(\alpha + \beta)$ .

[10 pts.]