Math 1271
Fall 2004 Final Exam Problems

This exam contains 12 multiple-choice problems, worth 8 points each, and 6 written problems, worth 16 - 18 points each, for a total of 200 points. (Problem #18 is missing from this set.)

1. Let $f(x) = (3x^2 + 5x - 6)^3$. Then $f'(1)$ is equal to

(A) 12
(B) $3(3 + 5 - 6)^2$
(C) $(3 + 5 - 6)^4$
(D) $3(3 + 5 - 6)^2(6 + 5)$
(E) 144

2. The tangent line to the curve $y = x^3 - 2x^2 + 2x + 1$ at the point (2,5) has equation

(A) $y - 5 = (3x^2 - 4x + 2)(x - 2)$
(B) $y = 5x/2$
(C) $y - 5 = (12 - 8 + 2)(x - 2)$
(D) $x - 2 = (12 - 8 + 2)(y - 5)$
(E) $y - 5 = -6(x - 2)$

3. $\lim_{x \to 2} \dfrac{x^2 - 4}{x^3 - 8}$ is equal to

(A) 0
(B) 1/2
(C) 1/3
(D) 2/5
(E) 2/3

4. Let $f(x)$ be defined by

$$f(x) = \begin{cases} |x - 2|, & \text{if } x < 3 \\ (x - 2)^2, & \text{if } 3 \leq x \leq 4 \\ x - 4, & \text{if } x > 4. \end{cases}$$

Then f is continuous

(A) except at $x = 2$;

(B) except at $x = 3$;

(C) except at $x = 4$;

(D) except at $x = 3$ and $x = 4$;

(E) except at $x = 2$, $x = 3$ and $x = 4$.

5. Let $f(x) = 2x^3 - 3x^2 - 12x$. Then

$$f'(x) = 6(x-2)(x+1) \text{ and } f''(x) = 6(2x-1).$$

Then the absolute maximum of $f(x)$ on the interval $[-2, 2]$ occurs

(A) at $x = -2$

(B) at $x = -1$

(C) at $x = 0$

(D) at $x = 2$

(E) nowhere

6. The equation $7x^2y^3 - 5xy^2 - 4y = 7$ defines y implicitly as a function of x. Find dy/dx.

(A) $\dfrac{14xy^3 + 5y^2}{4 - 21x^2y^2 - 10xy}$

(B) $\dfrac{5y^2 - 14xy^3}{21x^2y^2 - 10xy - 4}$

(C) $\dfrac{5y^2 + 14xy^3}{21x^2y^2 - 10xy - 4}$

(D) $(7x^2y^3 - 5xy^2)/4$

(E) 0

7. Suppose that $f(x)$ is a function with first derivative $f'(x) = \dfrac{x^2 - 3x}{(x-1)^2}$. Then $f(x)$ is increasing on

(A) $(-\infty, 1)$ and $[3, \infty)$

(B) $[0, 1)$ and $[3, \infty)$

(C) $(-\infty, 0]$ and $(1, 3]$

(D) $(-\infty, 0]$ and $(1, \infty)$

(E) $(-\infty, 0]$ and $[3, \infty)$

8. Let $f(x) = (x+2)e^x$. Then, using the Mean Value Theorem, we can conclude that there is at least one number c between 1 and 4 such that $f'(c)$ is equal to

(A) $2e^4 - e$

(B) $3e^4 - (3/2)e$

(C) $3e^4 + (3/2)e$

(D) $6e^4 - 3e$

(E) $6e^4$

9. $\int \frac{x^{1/2}+x}{x^{5/2}}dx =$

(A) $-\frac{1}{x} - \frac{2}{\sqrt{x}} + C$

(B) $\frac{\frac{3}{2}x^{3/2}+\frac{1}{2}x^2}{\frac{7}{2}x^{7/2}+C}$

(C) $\frac{3}{x^3} + \frac{5}{2x^{5/2}} + C$

(D) $\frac{1}{x} + \frac{2}{\sqrt{x}} + C$

(E) $-\frac{1}{x} - \frac{1}{2\sqrt{x}} + C$

10. Let $f(x) = \int_2^x \sqrt{7t^2+8}\,dt$. Then $f'(2) =$

(A) 0

(B) 2

(C) 6

(D) $\frac{7}{3}$

(E) $\frac{1}{12}$

11. The substitution $x = u^2$ turns $\int_2^3 \tan\sqrt{x}dx$ into

(A) $\int_{\sqrt{2}}^{\sqrt{3}} \tan u \; du$

(B) $\int_{\sqrt{2}}^{\sqrt{3}} 2u \tan u \; du$

(C) $\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{2} u \tan u \; du$

(D) $\int_4^9 \tan u \; du$

(E) $\int_4^9 2u \;\tan u \; du$

12. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{2}$

(D) 2π

(E) $\frac{\pi}{6}$

13.(16 points)

a) If $x^2 + y^2 = 2$, find $\frac{dy}{dx}$.

b) Find an equation of the tangent to the circle $x^2 + y^2 = 2$ at the point $(1, -1)$.

14.(17 points) Show that the equation $5x - 7 - \sin x = 0$ has exactly one real root.

15.(17 points) A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$. Find the distance traveled during the time period $1 \leq t \leq 4$.

16.(18 points) Find the area of the largest rectangle that can be inscribed in a semicircle of radius 1. Explain why your answer is an absolute maximum.

17.(18 points) Consider the function

$$f(x) = \frac{x^2}{x^2 - 4}.$$

We have

$$f'(x) = \frac{-8x}{(x^2 - 4)^2} \text{ and } f''(x) = \frac{8(4 + 3x^2)}{(x^2 - 4)^3}.$$

a) Find the domain of $f(x)$.

b) Determine the x-intercept and y-intercept of $y = f(x)$.

c) Determine the horizontal and vertical asymptotes of $y = f(x)$.

d) Determine the critical points, intervals of increase or decrease of $f(x)$.

e) Determine the concavity intervals of $f(x)$ and points of inflection.

f) Sketch the curve $y = f(x)$.

(Problem #18 is missing)