Math 1272
Fall 2004 Final Exam Problems

This exam contains 14 multiple-choice questions, worth 7 points each, and 6 written problems, worth 10 - 23 points each, for a total of 200 points.

(On the last page is a table of formulas you may find helpful.)

1. $\int_0^1 xe^{4x}dx =$

(A) $\frac{3}{4}e^4 + \frac{1}{4}$ **(B)** $\frac{3}{16}e^4 + \frac{1}{16}$ **(C)** $\frac{5}{16}e^4 + \frac{1}{16}$ **(D)** $\frac{3}{16}e^4 - \frac{1}{16}$

(E) $\frac{3}{4}e^4 - \frac{1}{4}$

2. In order to find an antiderivative of $\frac{9x^4 + 15x^3 + 33x^2 + 14x + 4}{(3x+2)^2(x^2+2)}$ we should rewrite it as a partial fraction decomposition which is in the form

(A) $\frac{A}{(3x+2)^2} + \frac{B}{x^2+2}$ **(B)** $\frac{Ax+B}{x^2+2} + \frac{C}{(3x+2)^2} + \frac{D}{3x+2}$ **(C)** $\frac{Ax}{x^2+2} + \frac{B}{(3x+2)^2} + \frac{C}{3x+2}$

(D) $\frac{A}{x^2+2} + \frac{B}{(3x+2)^2} + \frac{C}{3x+2}$ **(E)** None of the above

3. A radioactive substance decays at a rate proportional to the amount of the substance. If there are 90 grams of the substance initially, and 40 grams remain after 4 years, how much is left after 7 years? Express your answer in grams.

(A) $40\sqrt{6}/9$ **(B)** 2.5 **(C)** $(40/7)e^{-90/4}$ **(D)** $90(4/9)^{7/4}$

(E) $90e^{-7/4}$

4. $\int_0^{\frac{\pi}{3}} \sec^4 x \tan x \, dx =$

(A) $16\sqrt{3}$ **(B)** $\frac{7}{36}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{15}{4}$ **(E)** 4

5. What is the area of the part of the plane that is bounded by the curve $r = \ln\theta$, $\theta = 1$ and $\theta = e$ given in polar coordinates?

(A) $\frac{e}{2} - 2$ **(B)** $\frac{e}{4} - 2$ **(C)** $\frac{e}{4} - 1$ **(D)** $\frac{e}{2} - 1$ **(E)** e

6. The integral $\int_1^\infty (x^{\frac{4}{3}} + 5)^\alpha dx$ is convergent with $\alpha =$

(A) $-\frac{1}{4}$ **(B)** $-\frac{e}{4}$ **(C)** $-\frac{\pi}{4}$ **(D)** $-\frac{3}{4}$ **(E)** $-\frac{1}{5}$

7. Find the center of mass of the semicircle plate with a radius of 10, centered at $(1, 0)$ and contained in $\{(x, y) | \ y \geq 0\}$.

(A) $(1, \frac{40}{3\pi})$ **(B)** $(1, \frac{40}{\pi})$ **(C)** $(\frac{40}{3\pi}, 1)$ **(D)** $(1, \pi)$ **(E)** $(\pi, 1)$

8. The series $\sum_{n=1}^\infty \frac{n^\alpha}{n^4+1}$ converges

(A) for all α **(B)** for all $\alpha \leq 3$ and no other value of α

(C) for all $\alpha < 3$ and no other value of α **(D)** for all $\alpha > 3$ and no other value of α

(E) for all $\alpha \geq 3$ and no other value of α

9. The sum of the geometric series $3 - \frac{15}{7} + \frac{75}{49} - \frac{375}{343} + \cdots$ equals

(A) $\frac{21}{15}$ **(B)** $\frac{21}{2}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{7}{2}$ **(E)** $\frac{7}{4}$

10. The volume of the parallelpiped determined by the vectors $< 1,0,0 >, < 1,2,-1 >$ and $< 0,1,1 >$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

11. The Taylor polynomial $T_5(x;0)$ of the function $f(x) = x\cos(x^2)$ is

(A) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ **(B)** $x - \frac{x^3}{2!} + \frac{x^5}{4!}$ **(C)** $x - \frac{x^5}{2}$ **(D)** $x + \frac{x^5}{2}$

(E) $x + x^3 + \frac{x^5}{2}$

12. The sum of the series $1 - \frac{2^2}{2!} + \frac{2^4}{4!} - \frac{2^6}{6!} + \cdots$ is equal to

(A) $\sin 2$ **(B)** $-\sin 2$ **(C)** $\cos 2$ **(D)** e^2 **(E)** e^{-2}

13. We have vectors $u = (1,2,3)$ and $v = (2,3,4)$. The following vector is orthogonal to u and v

(A) $(1,-2,0)$ **(B)** $(-2,1,0)$ **(C)** $(-2,0,1)$ **(D)** $(1,2,1)$

(E) $(1,-2,1)$

14. The distance between the parallel planes $x + y + z = 1$ and $x + y + z = 10$ is

(A) 9 **(B)** $3\sqrt{3}$ **(C)** 3 **(D)** 10 **(E)** $\sqrt{2}$

15. (20 pts) Compute each of the following integrals:

(a) $\int \sin^3 2x \, dx$ (b) $\int \frac{1}{x + x \ln x} dx$

16. (10 pts) Apply the Trapezoidal Rule to approximate $\int_0^\pi \sin(x^2) dx$ using $n = 6$. Write your answer as a sum of sine functions. Write down any general formulas you are using.

17. (23 pts) Solve the following differential equations:

(a) $y' = x^2 y$ What is the solution of (a) which satisfies $y(1) = 3$?

(b) $y' = x \sin(x^2)$

18. (19 pts) For what values of x does the series

$$\sum_{n=0}^{\infty} (-1)^n n 4^n x^n / (n^2 + 1)$$

converge? Carefully explain your reasoning, including all tests you are using.

19. (20 pts) Do the following series converge? Explain why or why not, including which tests you are using.

(a) $\sum_{n=8}^{\infty} 1/\sqrt{n+4}$ (b) $\sum_{n=3}^{\infty} \frac{1}{n(\log n)^2}$

20. (10 pts) What is the sine of the angle between the planes $x + y + 3z = 3$ and $2x + 2y + z = 5$?

Some Formulas

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin^2(A) = \frac{1}{2}(1 - \cos 2A)$$

$$\cos^2(A) = \frac{1}{2}(1 + \cos 2A)$$

$$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin^2 A + \cos^2 A = 1$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\cot^2 A + 1 = \csc^2 A$$