Math 1272
Fall 2005 Final Exam Problems

This exam contains 12 multiple-choice questions, worth 12 points each, and 6 written problems, worth 20 to 30 points each, for a total of 300 points.

1) Partial fraction decomposition of $\frac{1}{(x^2+1)(x-1)^2}$ should be looked for in the form

(A) $\frac{A}{x^2+1} + \frac{B}{(x-1)^2}$

(B) $\frac{Ax+B}{x^2+1} + \frac{C}{(x-1)^2}$

(C) $\frac{Ax}{x^2+1} + \frac{Bx}{(x-1)^2} + \frac{C}{x-1}$

(D) $\frac{Ax+B}{x^2+1} + \frac{Cx}{(x-1)^2} + \frac{D}{x-1}$

(E) $\frac{Ax+B}{x^2+1} + \frac{C}{(x-1)^2} + \frac{D}{x-1}$

2) The integral $\int_1^\infty (x^{\frac{4}{3}}+5)^\alpha \, dx$ is convergent with $\alpha =$

(A) $-\frac{1}{4}$

(B) $-\frac{e}{4}$

(C) $-\frac{\pi}{4}$

(D) $-\frac{3}{4}$

(E) $-\frac{1}{5}$

3) $\int_0^{\frac{\pi}{4}} x\cos x \, dx =$

(A) $\frac{\pi\sqrt{2}}{8} - \frac{\sqrt{2}}{2} + 1$

(B) $\frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1$

(C) $\frac{\pi\sqrt{2}}{4} + \sqrt{2} - 1$

(D) $\frac{\pi\sqrt{2}}{4} - \sqrt{2} + 1$

(E) $\frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}$

4) $\int_0^{\frac{\pi}{3}} \tan^2 x \, dx =$

(A) $1+\sqrt{3}$

(B) $\sqrt{3}-1$

(C) $\frac{\sqrt{3}}{9} - 1$

(D) $\sqrt{3}$

(E) $\sqrt{3} - \frac{\pi}{3}$

5) The slope of the tangent line to the curve $x = t^3 + 5t + 1, y = 2t^3 - 2t + 1$, at the point corresponding to $t = 1$, equals

(A) 7

(B) 1/7

(C) 1/2

(D) 2

(E) −2/5

6) The area of the surface generated by rotating the curve $x = t^2 \sin 2t$, $y = t^2 \cos 2t$, $0 \le t \le \frac{\pi}{4}$, about the x-axis, is given by

(A) $4\pi \int_0^{\pi/4} (t^3 \cos 2t)\sqrt{1+t^2}\, dt$

(B) $4\pi \int_0^{\pi/4} (t^3 \sin 2t)\sqrt{1+t^2}\, dt$

(C) $4\pi \int_0^{\pi/4} t\sqrt{1+t^2}\, dt$

(D) $4\pi \int_0^{\pi/4} (t^5 \cos^2 2t)\sqrt{1+t^2}\, dt$

(E) $4\pi \int_0^{\pi/4} (t^5 \sin^2 2t)\sqrt{1+t^2}\, dt$

7) The Cartesian coordinates of a point are $(-5\sqrt{3}, 15)$. Its polar coordinates (r, θ), with $r > 0$, and $0 \le \theta < 2\pi$, are

(A) $(5(3+\sqrt{3}), \frac{\pi}{3})$

(B) $(10\sqrt{3}, \frac{\pi}{3})$

(C) $(10\sqrt{3}, \frac{2}{3}\pi)$

(D) $(10\sqrt{3}, \frac{4}{3}\pi)$

(E) $(10\sqrt{3}, \frac{5}{6}\pi)$

8) The Taylor polynomial $T_5(x; 0)$ of the function $f(x) = x\cos(x^2)$ is

(A) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$

(B) $x - \frac{x^3}{2!} + \frac{x^5}{4!}$

(C) $x - \frac{x^5}{2}$

(D) $x + \frac{x^5}{2}$

(E) $x + x^3 + \frac{x^5}{2}$

9) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n^2+1)^p}$ is

(A) absolutely convergent for $p \geq \frac{1}{2}$

(B) absolutely convergent for $p > 1$ and conditionally convergent for $0 < p \leq 1$

(C) absolutely convergent for $0 < p \leq \frac{1}{2}$ and conditionally convergent for $p > \frac{1}{2}$

(D) divergent for all values of p

(E) absolutely convergent for $p > \frac{1}{2}$ and conditionally convergent for $0 < p \leq \frac{1}{2}$

10) The volume of the parallelipiped determined by the vectors $\langle 1,1,0\rangle, \langle 1,0,1\rangle$ and $\langle 0,1,1\rangle$ is

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

11) The sum of the geometric series $3 - \frac{15}{7} + \frac{75}{49} - \frac{375}{343} + \cdots$ equals

(A) $\frac{21}{15}$

(B) $\frac{21}{2}$

(C) $\frac{7}{12}$

(D) $\frac{7}{2}$

(E) $\frac{7}{4}$

12) Let $y(x)$, $x > 0$ satisfy $xy' = -y$ and $y(5) = 2$. Then $y(1) =$

(A) 10

(B) 5

(C) $\frac{2}{5}$

(D) 0

(E) 2

13) (30 points) Decide (with justification) if the series $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$, $\sum_{n=1}^{\infty} \left(\frac{\sin 1}{n}\right)^2$ and $\sum_{n=1}^{\infty} \sin^2\left(\frac{1}{n}\right)$ are convergent or divergent.

14) (25 points) Calculate the integral $\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}}\, dx$ (or prove that it diverges).

15) (20 points) Calculate the integral $\int x^3\sqrt{1-x^8}\, dx$.

16) (30 points)

(a) Sketch the curve given by the polar equation $r = 1 + 2\sin\theta$.

(b) Set-up, but do not evaluate an integral giving the length of the curve in part (a).

(c) Find the area enclosed by the inner loop of the curve in part (a).

17) (25 points)

(a) Write an equation for the plane through the point $(1, 2, 3)$ and which contains the line $x = 3t$, $y = 1 + t$, $z = 2 - t$.

(b) Find symmetric equations for the line through (1,2,3) which is perpendicular to the plane in part (a).

18) (26 points)

(a) Determine the radius of convergence R for the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{(2n+1)(2n+2)3^{2n+1}}$$

(b) If $f(x)$ is the function defined as the sum of the series in part (a) for $|x| < R$, find the exact value of $f'(\sqrt{3})$.