Math 1272
Spring 2005 Final Exam Problems

This exam contains 14 multiple-choice problems, worth 6 points each, and 6 written problems, worth 15 - 21 points each, for a total of 200 points.

1. Let
$$a_n = \frac{3+5n^2}{n+n^2}.$$
Which of the following statements on the sequence $\{a_n\}_{n=1}^{\infty}$ is correct:

   (a) $\lim_{n\to\infty} a_n = 0$;
   (b) $\lim_{n\to\infty} a_n = 3$;
   (c) $\lim_{n\to\infty} a_n = 4$;
   (d) $\lim_{n\to\infty} a_n = 5$;
   (e) The sequence is divergent.

2. $\int_0^\infty xe^{-2x}dx =$

   (a) 0;
   (b) $\frac{1}{4}$;
   (c) $\frac{1}{2}$;
   (d) 2;
   (e) ∞.

3. The partial fraction decomposition of the rational function
$$\frac{x^4+3x^2+2x+2005}{(x^2-1)(x^2+1)^2}$$
has the form

   (a) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x^2+1} + \frac{D}{(x^2+1)^2}$;
   (b) $\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$;
   (c) $\frac{A}{x^2-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$;
   (d) $\frac{A}{x^2-1} + \frac{B}{x^2+1} + \frac{C}{(x^2+1)^2}$;
   (e) $\frac{A}{x^2-1} + \frac{B}{(x^2+1)^2}$.

4. $\sum_{n=0}^{\infty} \frac{(\ln 5)^n}{n!} =$

(a) 0;

(b) $\ln 5$;

(c) 5;

(d) e^5;

(e) ∞.

5. Consider the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}.$$

Which of the following statement is **incorrect**:

(a) It converges when $p > 0$;

(b) It diverges when $p < 1$;

(c) It is absolutely convergent when $p > 1$;

(d) It is conditionally convergent when $0 < p < 1$;

(e) It diverges when $p = 0$.

6. The distance between two parallel planes

$x - 2y + 2z = 2005$ and $x - 2y + 2z = 2000$ is

(a) $\frac{5}{3}$;

(b) 3;

(c) 5;

(d) 4005;

(e) None of the above.

7. The area of the region bounded by the polar curve $r = e^\theta$ and by the rays $\theta = 0$ and $\theta = 1$ is

(a) $\frac{1}{4}$;

(b) $\frac{1}{4}(e^2 - 1)$;

(c) $\frac{1}{4}e^2$;

(d) $\frac{1}{2}(e - 1)$;

(e) None of the above.

8. Consider the parametric curve

$$\begin{cases} x = t + \ln t; \\ y = 1 + t^2. \end{cases}$$

The slope of the tangent line at point $(1, 2)$ is

(a) 1;
(b) 2;
(c) 5;
(d) 0;
(e) $\frac{1}{2}$.

9. The curve $y = \sqrt{4 - x^2}$, $0 \leq x \leq 1$ is rotated around the x-axis. The area of the resulting surface is

(a) 2;
(b) 4;
(c) 4π;
(d) 2π;
(e) π.

10. Which of the following series is **divergent**:

(a) $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$;
(b) $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$;
(c) $\sum_{n=0}^{\infty} \frac{n^2+2n+10}{3n^5+n^2+1}$;
(d) $\sum_{n=0}^{\infty} \frac{n^3+15}{n^5+1}$;
(e) $\sum_{n=0}^{\infty} \frac{n+3}{n^2+2n+1}$.

11. $\int \sec^4 x \tan^2 x dx =$

(a) $\sec^6 x + C$
(b) $\sec^4 x + \tan^2 x + C$;
(c) $\sec^4 x \tan^2 x + C$;
(d) $\frac{1}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C$;
(e) $\frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C$.

12. Point A has a spherical coordinate $\left(4, \frac{\pi}{3}, \frac{\pi}{6}\right)$. Then its cartesian coordinate is

(a) $\left(1, 2\sqrt{3}, \sqrt{3}\right)$;
(b) $\left(1, \sqrt{3}, 2\sqrt{3}\right)$;
(c) $\left(\sqrt{3}, 3, 2\right)$;
(d) $\left(3, \sqrt{3}, 2\right)$;
(e) $(1, 2, 3)$.

13.

$$\sum_{n=0}^{\infty} \frac{(-3)^{n+2}}{2^{2n}} =$$

(a) $\frac{36}{7}$;
(b) 36;
(c) $\frac{4}{7}$;
(d) 4;
(e) None of the above.

14. Consider the infinite series

$$\sum_{n=0}^{\infty} (-1)^n \frac{\ln(n+3)}{n+3}.$$

Which statement is **incorrect**:

(a) It is an alternating series;
(b) $\lim_{n \to \infty} \frac{\ln(n+3)}{n+3} = 0$;
(c) $\frac{\ln(n+3)}{n+3}$ is monotone decreasing in n;
(d) The series converges;
(e) The series is absolutely convergent.

15. (20 points) Evaluate the integrals:

(a) (10 points)

$$\int 2x^3 \sin\left(1+x^2\right) dx;$$

(b) (10 points)

$$\int_0^4 \frac{1-\sqrt{x}}{1+\sqrt{x}} dx.$$

16. (15 points) Consider the differential equation

$$\left(x^2+1\right) y' = y - 3.$$

(a) (10 points) Find the general solutions to the equation;

(b) (5 points) Find a solution satisfying $y(0) = 2005$.

17. (20 points) Consider curve $y = f(x)$, $0 \le x \le 2$, where

$$f(x) = \int_0^x \sqrt{e^{2t}-1}\, dt.$$

(a) (5 points) Find $\frac{dy}{dx}$ using Fundamental theorem in Calculus;

(b) (15 points) Find the length of the curve.

18. (20 points) Let

$$f(x) = e^{-x^2} \sin x^3.$$

(a) (15 points) Find the first five nonzero terms of the Maclaurin series of f.

(b) (5 points) Using the coefficients of the series to find $f^{(7)}(0)$.

19. (20 points) Find the radius and interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{2^n (x+1)^n}{\sqrt{n+1}}.$$

Specify the test you use and show your reasoning.

20. (21 points) Let $A(2,3,4)$, $B(3,4,2)$, $C(4,2,3)$ be three points in the space.

(a) (7 points) Find the angle between vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$;

(b) (7 points) Find the area of the triangle ABC;

(c) (7 points) Find the volume of the parallelepiped determined by the vectors $\overrightarrow{OA}$ $\overrightarrow{OB}$ and $\overrightarrow{OC}$.