Math 1051
Spring 2003 Final Exam Problems

This exam contains 15 multiple-choice questions, worth 3 points each, and 6 written problems, worth 9 points each (10 for #5), for a total of 100 points.

M1. After simplification, the expression $x+1-\dfrac{x^2+2x}{x+1}$ reads

a) x^2+2x;

b) 1;

c) $1-x^2-2x$;

d) 2;

e) $1/(x+1)$.

M2. Multiply polynomials

$$(x-a)(x^3+ax^2+a^2x+a^3).$$

The result after simplification is

a) $x^4+ax^3+a^2x^2+a^3x$;

b) $x^2+2ax+a^2$;

c) $-ax^3-a^2x^2-a^3x-a^4$;

d) x^4-a^4;

e) none of the above.

M3. The expression $x^{1/4}y^{-3/2}z^{1/6}$ is equal to (assuming that $x,y,z>0$)

a) $\dfrac{1}{4}x-\dfrac{3}{2}y+\dfrac{1}{6}z$;

b) $-\dfrac{3}{48}xyz$;

c) $\dfrac{\sqrt[4]{x}\sqrt[6]{z}}{\sqrt{y^3}}$;

d) $x^{1/4}z^{1/6}-y^{3/2}$;

e) none of the above.

M4. The Least Common Multiple of the polynomials

$$(x+1)(x-2)^2(x+3), \qquad (x+1)^2(x-2), \qquad (x+3)^3(x+1)$$

is

a) $(x+1)^2(x-2)^2(x+3)^3$;

b) $(x+1)^4(x-2)^3(x+3)^4$;

c) $(x+1)(x-2)(x+3)$;

d) $(x+1)^4+(x-2)^3+(x+3)^4$;

e) $(x+1),(x-2),(x+3)$.

M5. Divide $x^5 - x + 1$ by $x^2 + 1$. The quotient after division is

a) $x^3 + x$;

b) $x^3 + x^2 + x + 1$;

c) $x^3 - x$;

d) $x^3 - 3$;

e) $x^3 - x^2 - x - 1$.

M6. Find the midpoint M of the segment P_1P_2, $P_1 = (3, 1)$, $P_2 = (-2, 4)$. The answer is

a) $M = (5/2, -3/2)$;

b) $M = (1/2, 5/2)$;

c) $M = (5/2, 1/2)$;

d) $M = (1/3, 2/3)$;

e) $M = (1, 1)$.

M7. If a leg of a right triangle has length 3 and the hypotenuse has length 4, then the other leg will have length

a) $\sqrt{5}$;

b) 5;

c) $\sqrt{7}$;

d) 7;

e) none of the above.

M8. Consider functions

$$f(x) = \frac{1}{x^2 + 4x + 1}, \qquad g(x) = \frac{1}{x} + 3.$$

Which from the following is equal to $\big(g \circ f\big)(x)$ after the simplification?

a) $\dfrac{1}{x^2 + 4x + 1} + 3$;

b) $\big(\frac{1}{x} + 3\big)\big(x^2 + 4x + 1\big)$;

c) $(x + 2)^2$;

d) $x^2 + 4x + 1$;

e) $\dfrac{x^2 + 4x + 1}{x} + 3$.

M9. If $(1, b)$ is a point on the graph of $y = x^2 - 1$, what is b?

a) 0;

b) -1;

c) 1;

d) $b^2 - 1$;

e) 2.

M10. The Average Rate of Change of the function $f(x) = 1 + x^2$ from $x = 1$ to $x = 3$ is

a) 1;

b) $f(0)$;

c) $f(x) - f(0)$;

d) 5;

e) 4.

M11. Which from the following is true about the rational function $R(x) = \dfrac{3x^4 - 2x^3 + x + 1}{x^3 - 1}$?

a) $R(x)$ has a horizontal asymptote $y = 0$;

b) $R(x)$ has an oblique asymptote $y = 3x - 2$;

c) $R(x)$ has an oblique asymptote $y = 1/x$;

d) $R(x)$ has a horizontal asymptote $y = 3$;

e) none of the above.

M12. The value of $\dfrac{\log_b(y^4)}{\log_b(a^2)}$ is equal to (assuming $a, b > 0$, $a, b \neq 1$, and $y > 0$).

a) $2 \log_a y$;

b) $\log_1(y^4/a^2)$;

c) $\log_b(\log_2(y^4))$;

d) $\log_b(y^4/a^2)$;

e) none of the above.

M13. The inverse function $f^{-1}(x)$ to the function $f(x) = 1/x$ is

a) x^2;

b) $1/\sqrt{x}$, $x > 0$;

c) $\sqrt{x}$, $x > 0$;

d) $1/x$, $x \neq 0$;

e) x.

M14. Which from the following statements is true?

a) $e^{\pi} = N$ is equivalent to $\ln N = \pi$;

b) $\pi^e = M$ is equivalent to $\ln M = \pi$;

c) $M^N = e$ is equivalent to $\ln N = \ln M$;

d) $Z^y = e$ is equivalent to $\ln Z = y$;

e) $W^e = y$ is equivalent to $\ln W = y$.

M15. The domain of the function $f(x) = \log_{1-x}(x + 1)$ is

a) $x \neq -1$ and $x \neq 1$;

b) $x > 1$ or $x < -1$;

c) $1 < x < -1$ and $x \neq 0$;

d) $x \neq -1$, $x \neq 0$, and $x \neq 1$;

e) $x > 0$.

1. Simplify the expression

$$1 - \cfrac{1}{1 + \cfrac{1}{1 - \cfrac{1}{x}}}$$

2. Determine the domain of the function

$$f(x) = \frac{1}{\sqrt{x-2}} + \ln(5-x).$$

3. Find the center and the radius of the circle

$$x^2 + y^2 - 6x + 8y = 0.$$

4. What function is finally graphed after the following transformations are applied to the graph of

$$f(x) = \sqrt{x} + x$$

a) reflect about Y-axis;

b) compress horizontally by a factor of 2;

c) shift left 1 unit.

5. Graph the rational function

$$R(x) = \frac{x^3 - 27}{x^2 - 2x - 3}$$

6. Solve logarithmic equation

$$\log_4(x+3) + \log_4 x = 1.$$