Math 1272
Fall 2001 Final Exam Problems

This exam contains 15 multiple-choice questions and
5 written problems, all worth 20 points each, for a total of 400 points.

1. $\int_0^{\frac{\pi}{4}} x \cos x \, dx =$

(A) $\frac{\pi\sqrt{2}}{8} - \frac{\sqrt{2}}{2} + 1$
(B) $\frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1$
(C) $\frac{\pi\sqrt{2}}{4} + \sqrt{2} - 1$
(D) $\frac{\pi\sqrt{2}}{4} - \sqrt{2} + 1$
(E) $\frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}$

2. $\int_0^{\frac{\pi}{3}} \tan^2 x \, dx =$

(A) $1 + \sqrt{3}$
(B) $\sqrt{3} - 1$
(C) $\frac{\sqrt{3}}{9} - 1$
(D) $\sqrt{3}$
(E) $\sqrt{3} - \frac{\pi}{3}$

3. $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx =$

(A) 1
(B) $\frac{1}{4}$
(C) $\frac{8}{15}$
(D) $\frac{2}{15}$
(E) $\frac{1}{15}$

4. Partial fraction decomposition of $\dfrac{15x^3 + 33x^2 + 14x + 4}{(3x+2)^2(x^2+2)}$ should be looked for in the form

(A) $\dfrac{A}{x^2+2} + \dfrac{B}{(3x+2)^2}$
(B) $\dfrac{Ax+B}{x^2+2} + \dfrac{Cx}{(3x+2)^2} + \dfrac{D}{3x+2}$
(C) $\dfrac{Ax+B}{x^2+2} + \dfrac{C}{(3x+2)^2} + \dfrac{D}{3x+2}$
(D) $\dfrac{Ax}{x^2+2} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{3x+2}$
(E) $\dfrac{Ax}{x^2+2} + \dfrac{Bx}{(3x+2)^2} + \dfrac{C}{3x+2}$

5. $\int_{\frac{4}{3}}^{\frac{3}{5}} \frac{x}{1-x^2}\,dx =$

(A) $\sin^{-1}\left(\frac{4}{5}\right)$
(B) $\ln(\frac{3}{4})$
(C) $\frac{4}{5}$
(D) $\ln(\frac{4}{3})$
(E) $\sqrt{23}$

6. $\int_0^{\frac{3}{2}} \frac{dx}{9+4x^2} =$

(A) $\frac{1}{18}$
(B) $\frac{\pi}{12}$
(C) $\frac{\pi}{24}$
(D) $\frac{\pi}{54}$
(E) $\frac{\pi}{36}$

7. The length of the curve $y = \ln x, 1 \le x \le 2$, is given by

(A) $\int_1^2 \frac{\sqrt{x^2+1}}{x}\,dx$
(B) $\int_1^2 \sqrt{\frac{x+1}{x}}\,dx$
(C) $2\pi \int_1^2 \ln x\,dx$
(D) $2\pi \int_1^2 \frac{\sqrt{x^2+1}}{x}\,dx$
(E) $2\pi \int_1^2 \sqrt{\frac{x+1}{x}}\,dx$

8. Solve $xy' = -y$ for $x > 0$ and $y(5) = 2$

(A) $y = 10/x$
(B) $y = 10 \ln \frac{1}{x}$
(C) $y = 5 \cos x$
(D) $y = 10 \ln x$
(E) None of the above

9. The area of the surface generated by rotating the curve $x = \frac{2}{3}t^{3/2} + t$, $y = \frac{2}{3}t^{3/2} - t$, $0 \le t \le 1$, about the x-axis is given by

(A) $2\sqrt{2}\pi \int_0^1 \left(\frac{2}{3}t^{3/2} - t\right) \sqrt{t+1}\, dt$
(B) $2\sqrt{2}\pi \int_0^1 \left(\frac{2}{3}t^{3/2} + t\right) \sqrt{t+1}\, dt$
(C) $4\pi \int_0^1 \left(\frac{2}{3}t^{3/2} - t\right) (t+1)\, dt$
(D) $4\pi \int_0^1 \left(\frac{2}{3}t^{3/2} + t\right) (t+1)\, dt$
(E) $2\pi \int_0^1 \left(\frac{2}{3}t^{3/2} + t\right) \left(\frac{2}{3}t^{3/2} - t\right) dt$

10. The Cartesian coordinates of a point are $(-5\sqrt{3}, 15)$. Its polar coordinates (r, θ), with $r > 0$, and $0 \le \theta < 2\pi$, are

(A) $(5(3+\sqrt{3}), \frac{\pi}{3})$
(B) $(10\sqrt{3}, \frac{\pi}{3})$
(C) $(10\sqrt{3}, \frac{2}{3}\pi)$
(D) $(10\sqrt{3}, \frac{4}{3}\pi)$
(E) $(10\sqrt{3}, \frac{5}{6}\pi)$

11. The series $\sum_{n=1}^{\infty} \frac{n^\alpha + 1}{n^4}$ converges

(A) for all α
(B) for all $\alpha \le 3$ and no other value of α
(C) for all $\alpha < 3$ and no other value of α
(D) for all $\alpha > 3$ and no other value of α
(E) for all $\alpha \ge 3$ and no other value of α

12. We have vectors $u = (1, 0, 3)$ and $v = (2, 3, 0)$. The following vector is orthogonal to u and v

(A) $(1, -2, 0)$
(B) $(-2, 1, 0)$
(C) $(3, -2, -1)$
(D) $(1, 2, 1)$
(E) $(1, -2, 3)$

13. The distance between the plane $x + 2y + 3z = 4$ and the origin is

(A) $\frac{4}{\sqrt{11}}$
(B) $\frac{4}{\sqrt{12}}$
(C) $\frac{4}{\sqrt{13}}$
(D) $\frac{4}{\sqrt{14}}$
(E) $\frac{4}{\sqrt{15}}$

14. The equation $-\frac{2x}{3} + x^2 + (-2+y)^2 + 8y - 2z + z^2 = \frac{26}{9}$ represents a sphere with

(A) center $(2, -\frac{1}{6}, 1)$ and radius $\sqrt{2}$
(B) center $(\frac{1}{3}, -2, 1)$ and radius 2
(C) center $(-\frac{1}{6}, 1, 0)$ and radius $\sqrt{2}$
(D) center $(\frac{1}{6}, -2, 1)$ and radius 2
(E) center $(\frac{1}{3}, 2, -1)$ and radius 2

15. Let $(4, \frac{\pi}{6}, \frac{\pi}{3})$ be given in spherical coordinates. What is it in rectangular coordinates?

(A) $(3, \sqrt{3}, 2)$
(B) $(\sqrt{3}, 3, 2)$
(C) $(2, \sqrt{3}, 3)$
(D) $(-2, \sqrt{3}, 3)$
(E) $(3, \sqrt{3}, -2)$

16. Does the series $\sum_{n=1}^{\infty} ne^{-n^2}$ converge? Justify your answer.

17. A curve is given parametrically by $x = t^3 - 3t^2,\ y = t^3 - 3t$.

a) Prove that $\frac{dy}{dx} = \frac{(t-1)(t+1)}{t(t-2)}$

b) Find the points at which the tangent line to the curve is horizontal.

c) Find the points at which the tangent line to the curve is vertical.

d) Find the intervals of the parameter t on which the curve rises.

e) Find the intervals of the parameter t on which the curve falls.

f) Sketch the curve.

18. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n \cdot (n^2+1)}$. Justify your answer.

19. How many terms of the Taylor series for $f(x) = \cos x$ centered at $a = 45^o$ (i.e. $a = \frac{\pi}{4}$) should be used to estimate $\cos 42^o$ (i.e. $\cos(\frac{\pi}{4} - \frac{\pi}{60})$) to within 0.0001?
(Recall Taylor's inequality: if $|f^{(n+1)}(x)| \leq M$, then $\left|f(x) - \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i\right| \leq \frac{M}{(n+1)!}(x-a)^{n+1}$)

20. Find the volume of the parallelopiped

$A(-1,2,0)$
$B(0,3,2)$
$C(-3,0,3)$
$D(-2,1,1)$