Math 1272
Spring 2003 Final Exam Problems

This exam contains 15 multiple-choice questions, worth 13 points each, and 7 written problems, worth 30 points each (#16 is 25 points), for a total of 300 points.

1. A radioactive substance obeys $dx/dt = kx$, where x is mass, where t is time in years, and where k is a negative constant. If there are 180 grams of the substance initially, and 80 grams remain after 8 years, then k is equal to

(A) $\ln(2/3)$
(B) $(1/2)\ln(2/3)$
(C) $(1/4)\ln(2/3)$
(D) $2\ln(2/3)$
(E) $-4/9$

2. Solve $xy' = y$ for $x > 0$ and $y(1) = 5$

(A) $y = 5x^2$
(B) $y = 5x^3$
(C) $y = 5x$
(D) $y = 5$
(E) None of the above

3. $\int_0^1 xe^{4(x-1)}\,dx =$

(A) $\frac{3}{4} + \frac{1}{4}e^{-4}$
(B) $\frac{3}{16} + \frac{1}{16}e^{-4}$
(C) $\frac{5}{16} + \frac{1}{16}e^{-4}$
(D) $\frac{3}{16} - \frac{1}{16}e^{-4}$
(E) $\frac{3}{4} - \frac{1}{4}e^{-4}$

4. $\int \frac{(-2)}{x^2+6x+8}\,dx =$

(A) $2\tan^{-1}(x+3) + C$
(B) $\ln|x+1| + \ln|x+3| + C$
(C) $\ln|x+4| - \ln|x+2| + C$
(D) $\ln|x+2| - \ln|x+4| + C$
(E) $\ln|x+1| - \ln|x+3| + C$

5. $\int \frac{x+2}{\sqrt{9-x^2}}\,dx =$

(A) $-\sqrt{9-x^2} + \arcsin 3x + C$

(B) $-3\sqrt{9-x^2} + 2\arcsin\frac{x}{3} + C$

(C) $-\frac{3}{2}\sqrt{9-x^2} + 4\arcsin\frac{x}{3} + C$

(D) $-\sqrt{9-x^2} + 2\arcsin\frac{x}{3} + C$

(E) $3\sqrt{9-x^2} + 3\arcsin\frac{x}{3} + C$

6. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x\,dx =$

(A) $-\frac{2}{3}$

(B) $\frac{2}{3}$

(C) 0

(D) 2

(E) $\frac{1}{2}$

7. The vectors $(\alpha^2, 2, -3)$ and $(9, 2, 4\alpha)$ are orthogonal (or perpendicular)

(A) for no real number α

(B) for exactly one real number α

(C) for exactly two (distinct) real numbers α

(D) for infinitely many real numbers α, but not for all

(E) for all real numbers α

8. The Cartesian coordinates of a point are $(4\sqrt{3}, 12)$. Its polar coordinates (r, θ), with $r > 0$, and $0 \leq \theta < 2\pi$, are

(A) $(8\sqrt{3}, \frac{2}{3}\pi)$

(B) $(8\sqrt{3}, \frac{1}{3}\pi)$

(C) $(6 - 2\sqrt{3}, \frac{5}{6}\pi)$

(D) $(48, \frac{2}{3}\pi)$

(E) $(4\sqrt{3}, \frac{5}{3}\pi)$

9. The sum of the geometric series $2 - \frac{6}{7} + \frac{18}{49} - \frac{54}{343} + \cdots$ equals

(A) $\frac{21}{8}$

(B) $\frac{21}{2}$

(C) $\frac{7}{3}$

(D) $\frac{7}{4}$

(E) $\frac{7}{5}$

10. The slope of the tangent line to the curve $x = 3t^3 + 6t^2 + 1, y = 7t^3 + 7$, at the point corresponding to $t = 1$, equals

(A) 5

(B) 4

(C) 3

(D) 2

(E) 1

11. The Taylor series of $x\cos(x^3)$ about the point $a = 0$ is

(A) $\sum_{k=0}^{\infty} (-1)^k \frac{x^{6k+1}}{(2k)!}$

(B) $\sum_{k=0}^{\infty} (-1)^k \frac{x^{6k}}{(2k)!}$

(C) $\sum_{k=0}^{\infty} (-1)^k \frac{x^{3k}}{(6k)!}$

(D) $\sum_{k=0}^{\infty} (-1)^k \frac{x^{4k+1}}{(4k)!}$

(E) $\sum_{k=0}^{\infty} (-1)^k \frac{x^{4k+2}}{(4k)!}$

12. Find the area of the region bounded by the curve $x = t^3$, $y = t^{10}$ and the lines $x = 1$, $y = 0$.

(A) $\frac{1}{13}$

(B) $\frac{2}{13}$

(C) $\frac{3}{13}$

(D) $\frac{4}{13}$

(E) $\frac{5}{13}$

13. The length of the curve $x = t^2 \sin 2t$, $y = t^2 \cos 2t$, $0 \leq t \leq 1$, equals

(A) $\frac{8}{3}\sqrt{2}$

(B) $\frac{2}{3}\sqrt{2}$

(C) $\frac{1}{3}(2\sqrt{2}-1)$

(D) $\frac{2}{3}(2\sqrt{2}-1)$

(E) $\frac{1}{3}(2\sqrt{2}+1)$

14. We have vectors $u = (0, 2, -1)$ and $v = (6, 1, -2)$. The following vector is orthogonal to u and v

(A) $(0, 2, 4)$

(B) $(1, 0, 3)$

(C) $(1, 3, 6)$

(D) $(1, 2, 4)$

(E) $(1, -2, 3)$

15. The convergence radius for the Maclaurin series of the function $f(x) = \frac{1}{3x^2+1} - \frac{1}{x+3}$ is

(A) $\frac{1}{\sqrt{3}}$

(B) 3

(C) $\frac{1}{3}$

(D) 1

(E) the series is convergent everywhere

16. (25 points) Evaluate the following integral:

$$\int_{-\infty}^{0} (x-1)e^x dx.$$

17. (30 points) Find the indefinite integral:

$$\int (\ln x)^2 dx.$$

18. (30 points) Find an equation of the plane through the point $(3, 1, 5)$ and parallel to the plane $x - y - 2z = 3$.

19. (30 points) Find the area of the surface obtained by rotating the part of the curve

$$y = \sqrt{100 - x^2}$$

between $x = 0$ and $x = 3$ around the x-axis.

20. (30 points) Consider the curve given in polar coordinates by the equation

$$r = 2\cos\theta + 2\sin\theta.$$

(a) Find an equation for the tangent line to the curve at the point where $\theta = \frac{\pi}{4}$.
(b) Sketch the region enclosed by this curve. USING INTEGRATION find the area enclosed by this curve.

21. (30 points) Consider the sequence a_k where

$$a_k = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} \left(\frac{k}{k+1}\right)^{2n+1}.$$

Show that the limit as $k \to \infty$ of a_k is $\frac{\pi}{4}$.
(HINT: express this limit as a definite integral.)

22. (30 points) Consider the series

$$\sum_{n=1}^{\infty} (-1)^n \cdot \left(\frac{1}{n}\right)^{1+\frac{1}{n}}$$

(a) Show that this series converges.
(b) Show that this series is not absolutely convergent.
(HINT: use the limit comparsion test.)